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Capturing Statistical Characteristics of High-Sigma Reliability Analysis Methods

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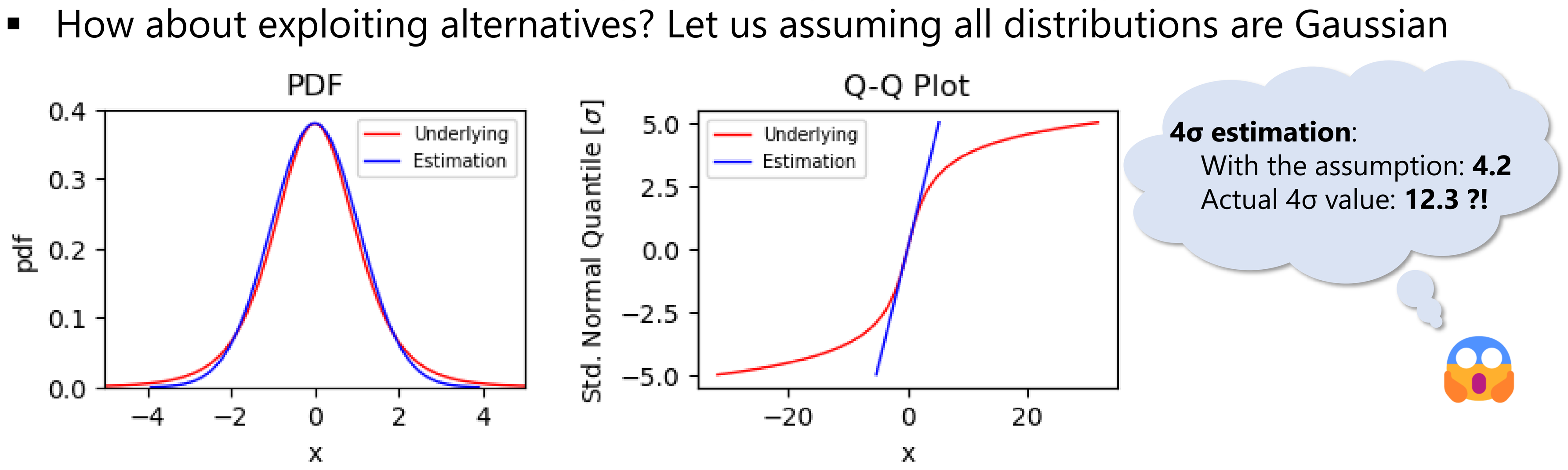
Abstract

Monte Carlo simulation is one of the most fundamental methods for analyzing circuit reliability to variations, including process, voltage, and temperature. However, it requires a large number of simulation samples, which makes it impractical in industry. Hopefully, many researchers have proposed various high-sigma reliability analysis methods over the last decade to overcome the limitation. On the other hand, evaluating their statistical characteristics has not been noticed well, despite of their importance. In this work , we address it considering their random nature. Specifically, we suggest a new evaluation strategy focused on capturing their statistical characteristics, with two metrics: mean squared error for estimation accuracy and 95% estimation interval for estimation pessimism. We also demonstrate its application to industry designs with some in-depth analysis as a case study. Through the experiment, we reveal a few statistical properties, such as the impact of the number of simulation samples on the estimation accuracy, optimistic/pessimistic estimation trend according to the underlying tail distribution, and so forth. The characteristics we found can be helpful for designers who want to derive the full performance of high-sigma reliability analysis methods.

1. Introduction: High-Sigma Reliability Analysis

- Many analog & mixed blocks are susceptible to various kinds of variations
- Monte Carlo (MC) method is simple, but requires lots of samples!
- How about exploiting alternatives? Let us assuming all distributions are Gaussian

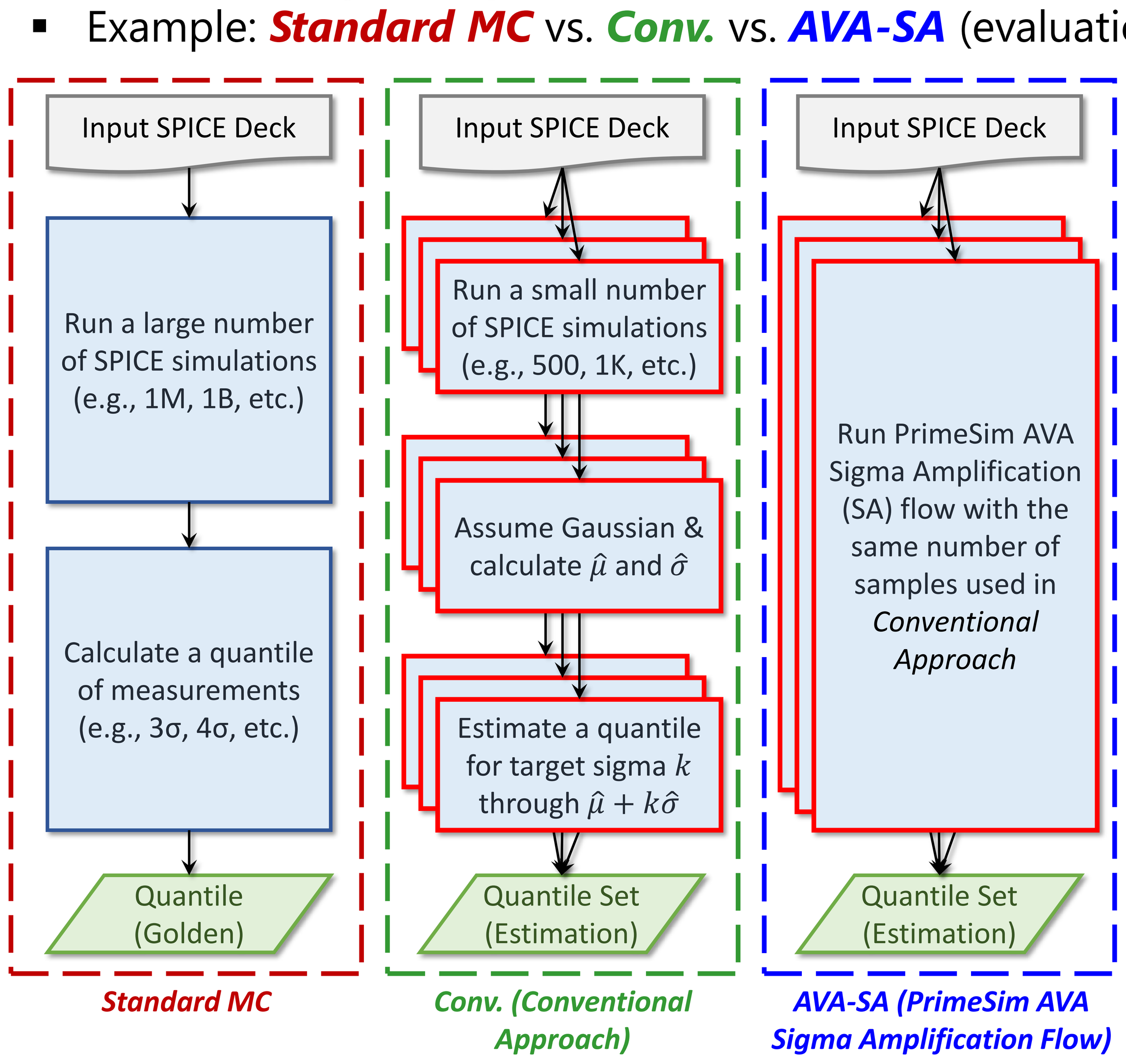


Note) Underlying: Student's t-distribution (df=5) / Estimation: Normal distribution (μ=0, σ=1.05)

High-sigma reliability analysis tools from EDA companies (PrimeSim AVA, Spectre FMC, etc.)
- Estimate high-sigma from running their methods **ONCE** and compare them with answers
- But their methods are usually based on **random algorithms**
  → How about **statistical behavior**?

2. Our Evaluation Strategy Details and Metrics

- Idea: **Running multiple sets with different random numbers/seeds**
- Example: ***Standard MC*** vs. ***Conv.*** vs. ***AVA-SA*** (evaluation scenario in this work)



① **Metric 1: Mean squared error (MSE)**

$$\mathrm{MSE}(\hat{\theta}) = \mathrm{E}_{\theta}\left[(\hat{\theta} - \theta)^2\right]$$

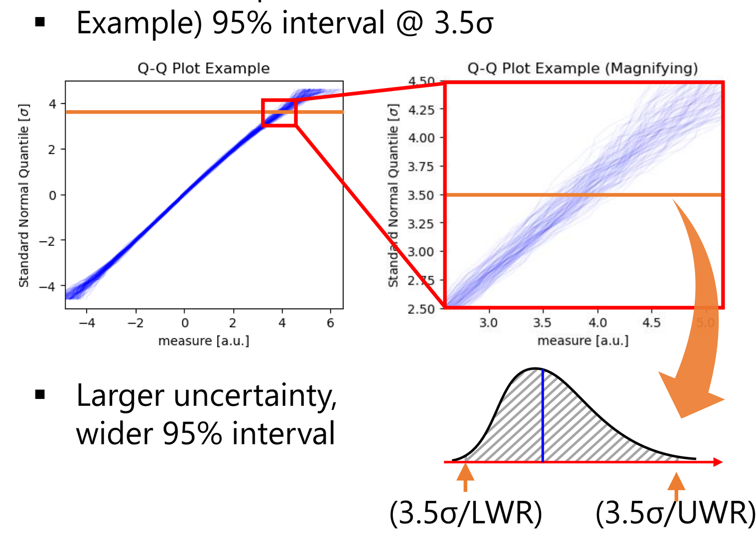
- MSE of an estimation $\hat{\theta}$ with respect to an unknown parameter θ
- Property)

$$\mathrm{MSE}(\hat{\theta}) = \underbrace{\left[\mathrm{Bias}(\hat{\theta}, \theta)\right]^2}_{\text{Bias term (bias error)}} + \underbrace{\mathrm{Var}_{\theta}(\hat{\theta})}_{\text{Variance term (variance error)}}$$

② **Metric 2: 95% estimation interval**

(95% estimation interval) = UWR − LWR

- LWR: 2.5% percentile of the distribution
- UWR: 97.5% percentile of the distribution
- Example) 95% interval @ 3.5σ



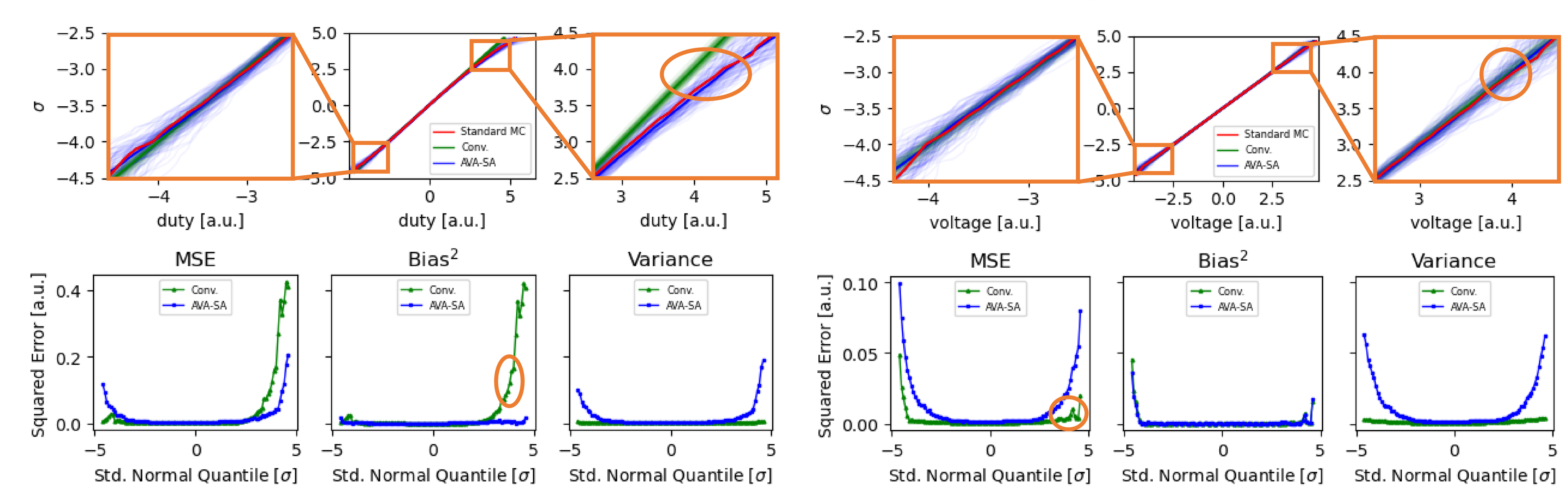
- Larger uncertainty, wider 95% interval

3. Experimental Results: Application of Our Strategy

Name	Function	#IRV	#MOSFET	#BJT	#Diode	#Res.	#Cap.	Meas.
CKT#1	I/O circuit (in DRAM)	35,241	8,760	12	2,560	97,005	164,202	duty
CKT#2	Voltage generator (in FLASH)	3,610	879	0	11	4,504	12,185	voltage

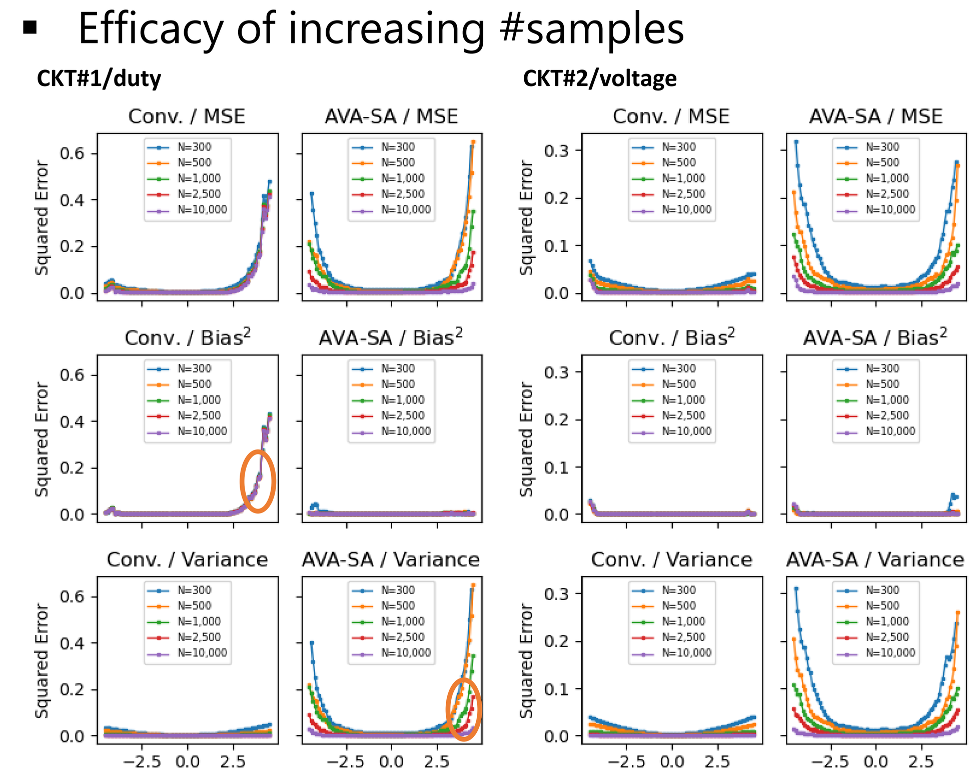
Note) #Sets: 100 / IRV: Independent Random Variables

- Quantile-Quantile (Q-Q) plots & MSE example (Left: CKT#1, Right: CKT#2, #Samples: 2.5K)



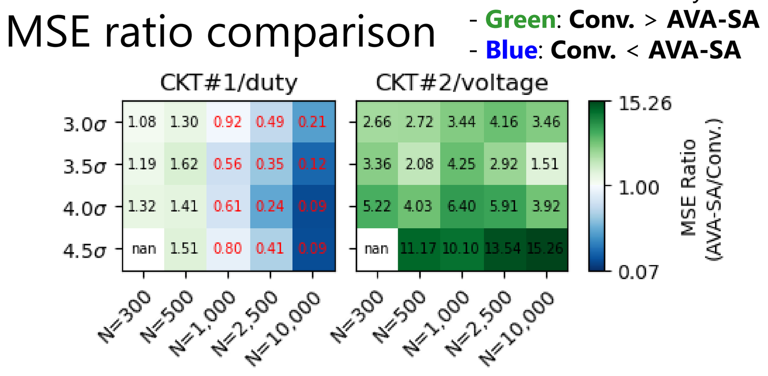
- Each thin green/blue line in Q-Q plots represents the estimation result from one set of running
- Thick green/blue lines in Q-Q plots represent the average of the estimation results over 100 sets
- Larger the gap with Gaussian distribution in the tail region, more accurate ***AVA-SA*** estimation

- Efficacy of increasing #samples



- MSE ratio comparison

※ Estimation accuracy:
- Green: Conv. > AVA-SA
- Blue: Conv. < AVA-SA

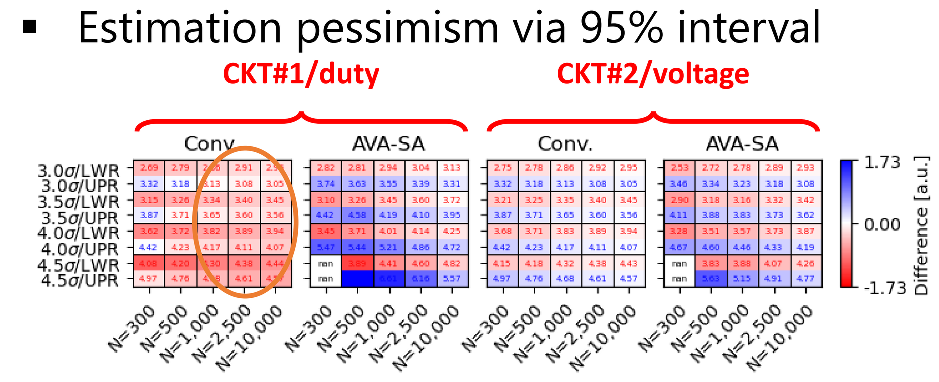


Note) N denotes #samples

- Estimation quality depends on *whether the tail region actually follows Gaussian or not*
- (CKT#1/duty:4.0σ with #samples≥1K) ***AVA-SA*** **reduces MSE to 0.31X on average (up to 0.09X)**, in comparison with that of ***Conv.***
- (CKT#2/voltage:4.0σ with #samples≥1K) **MSE of *AVA-SA* is larger than that of *Conv.* by 5.41X**, on average

- Almost not effective to bias term reduction while significantly effective for variance term reduction
- ***Conv.***: Bias term dominant vs. ***AVA-SA***: Variance term dominant
  → Increasing #samples is not a good solution for ***Conv.***, but has remarkable impact on ***AVA-SA***

- Estimation pessimism via 95% interval



- Blue/Red: Pessimistic/optimistic estimation
- (CKT#1/duty) ***Conv.*** estimates optimistically for the most cases, while ***AVA-SA*** estimates pessimistically for all the cases

Note) N denotes #samples

Summary & Limitation

- **In this work,**
  1. **we proposed the evaluation strategy of high-sigma reliability analysis methods for capturing its statistical characteristic using two metrics:**
     ① MSE for estimation accuracy and ② 95% inference interval for the amount of estimation pessimism;
  2. **we demonstrate the application of our strategy to industry designs with in-depth analysis.**
- Through the experiment, we revealed a few statistical properties of the methods during its evaluation.
- The characteristics might be helpful for designers who want to derive the full performance of the methods.
- **Limitation:** Not applicable to the high-sigma reliability analysis methods restricting a user's manipulation, e.g., Synopsys PrimeSim AVA – HSMC flow.